Mathematical and Computational Sciences

University of Toronto Mississauga

MAT132H5 - Differential Calculus for Life Sciences

Term Test 2 - November 8, 2019 - Version AUFW

Aids Permitted: None

Time Allotted: 110 minutes

Instructions

- Please have your student card ready for inspection, turn off all cellular phones, and read all the instructions carefully.
- This test contains two parts: Part A (36 marks) contains **nine** short questions, and Part B (64 marks) contains **four** questions. All answers are to be given in this booklet.
- Please **do not write anything on the QR codes** that appear at the top of each page.
- Check that this test has **12 pages**, including this cover page.
- There is a formula sheet on page 11, and space for rough work. You can also use page 10 and 12 for rough work. You may tear off the formula sheet, but you must **submit it** together with the rest of the test.
- Make sure to provide **exact answers**, using symbols such as $\sqrt{\ }$, e and π, if needed.

GOOD LUCK!

PART A (36 marks)

In this part, **clearly indicate your final short answer in the appropriate box**.
You must show your work (if any), even though **only the final answer will be graded**.
Simplify your answers **as much as possible**. Each question is worth **4 marks**.

1. Let $f(x)=\dfrac{3+x}{9-x^2}$. Find $\lim\limits_{x\to -3^-} f(x)$, $\lim\limits_{x\to 3^+} f(x)$ and $\lim\limits_{x\to +\infty} f(x)$.

   Your answer should be **a number**, $+\infty$, $-\infty$ or **DNE** if the limit does not exist.

Answer for 1.

$\lim\limits_{x\to -3^-} f(x) =$	$\lim\limits_{x\to 3^+} f(x) =$	$\lim\limits_{x\to +\infty} f(x) =$

2. Suppose that $g(x)$ is a differentiable function, satisfying $g(3)=g'(3)=4$. **Find the equation of the tangent line** to the graph of $g(x)$ at $x=3$. Write your answer in the form $y=mx+b$.

Answer for 2.

Continued on the next page...

---

3. Which of the following limits represents **the derivative** of $y=\ln x$ at $x=1$?

   Copy the correct limit to the box below.

$$\lim_{z\to 1}\frac{\ln z}{z} \quad ; \quad \lim_{h\to 0}\frac{\ln(1+h)}{h} \quad ; \quad \lim_{h\to 0}\frac{\ln h}{h-1} \quad ; \quad \lim_{z\to 1}\frac{\ln(1+z)}{z-1} \quad ; \quad \lim_{h\to 0}\frac{\ln(h-1)}{1+h}$$

Answer for 3.

4. If $f(x)=\dfrac{e^x}{\sqrt{x}}$, what is $f'(1)$? Simplify your answer, and express it using e , if needed.

Answer for 4.

$f'(1) =$

5. Calculate **the second derivative** of $g(x)=\sin(x^2)$ at $x=\sqrt{\pi}$. Use π in your answer, if needed.

Answer for 5.

Continued on the next page...

6. Consider the function $y=\sqrt{x^{1-c}}+\ln x$ (where c is a constant).

   If $\dfrac{dy}{dx}=\dfrac{7}{6}$ when $x=1$, what is **the value of the constant** c?

Answer for 6.

$c=$

7. A ball is dropped from a tall building. The height of the ball (in meters) at time t (in seconds) is given by $h(t)=320-5t^2$ (for $t\geq 0$). **How fast is the ball traveling when it hits the ground**?

Answer for 7.

Continued on the next page...

8. Find **the slope of the tangent line** to the curve $e^x - 2y = \cos y - 2x$ at $(0\,,0)$.

Answer for 8.

9. Suppose that a function $f(x)$ has an inverse.
   If the graph of the inverse function $f^{-1}(x)$ passes through the point $(-2\,,3)$, and has slope 5 at that point, **what are** $f(3)$ **and** $f'(3)$?

Answer for 9.

$f(3) =$ $\qquad\qquad$ $f'(3) =$

Continued on the next page...

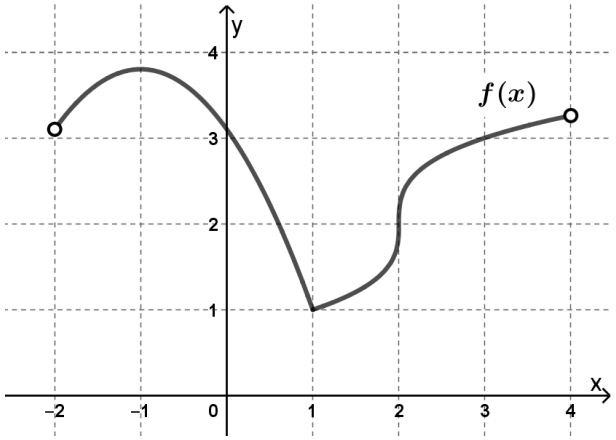
PART B (64 marks)

In this part you are required to provide **full solutions** and to **show all your work**.
A correct answer obtained with false reasoning or with no reasoning will not receive any marks.
Each question is worth **16 marks**.

1. Find the **equations** of all the **horizontal and vertical asymptotes** of the function $g(x) = \dfrac{\sqrt{8x}}{\sqrt{2x} - 3}$, if there are any. Show and explain your work.

Continued on the next page...

2. Here is the graph of a function $f(x)$ for $-2<x<4$. **Answer the questions below**.



(a) There are two x-values where $f(x)$ **exists** but has **no derivative**. What are they? Explain **why there is no derivative** at each of those values.

(b) If we drew the graph of **the derivative** $f'(x)$, what would we see at $x=1$? A corner? A jump? A hole? Something else? **Explain**.

Continued on the next page...

3. Let $h(x)=x^{\tan^{-1}x}$.

(a) Calculate $h'(x)$. Make sure your final answer is given in terms of x only.

(b) Find the **x-intercept** of the line that is **tangent** to the graph of $h(x)$ at $x=1$.
Express your answer using π , if needed.

Continued on the next page...

---

4. Use **the definition of the derivative as a limit** (i.e., first principles) to calculate the derivative of $g(x)=\sqrt{x^2+7}$ at $x=3$. **Show your work, and <u>do not</u> use any differentiation rules**.

Continued on the next page...

A BLANK PAGE OF EXTRA SPACE

DO NOT TEAR OFF THIS PAGE!

Continued on the next page...

FORMULA SHEET

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

$\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\sin\left(\dfrac{\pi}{4}\right) = \cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$

$\sin\left(\dfrac{\pi}{3}\right) = \cos\left(\dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

$\sin\left(\dfrac{\pi}{6}\right) = \cos\left(\dfrac{\pi}{3}\right) = \dfrac{1}{2}$

$\tan\left(\dfrac{\pi}{4}\right) = 1$

$[\log_a(x)]' = \dfrac{1}{x \cdot \ln a}$ (for $0 < a \neq 1$)

$(a^x)' = a^x \cdot \ln a$ (for $a > 0$)

Note: If you detach the formula sheet, place it at the back of your test and submit it as well. **Do not** tuck it into your test paper.

SPACE FOR ROUGH WORK

Continued on the next page...

SPACE FOR ROUGH WORK

THE END.