The Robert Gillespie ACADEMIC SKILLS CENTRE

Limits as x approaches a Real Number

Limits

Roughly speaking, finding the limit involves examining the behaviour of a function $f(x)$ as x approaches a real number a that may or may not be in the domain of f, that is,

$$\lim_{x \to a} f(x) = L, \text{ where } L \text{ is a real number,}$$

then it is said that the limit exists. Otherwise, the limit does not exist.

> **It is important to remember that limits describe the behaviour of a function near a particular point, but not necessarily at the point itself!**

Computation of Limits in Some Cases

Case 1. Direct substitution rule.

Example. Find $\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$.

Solution. By substituting $x = -2$,

$$\lim_{x \to -2} \frac{x^3 + 2x^2 - 1}{5 - 3x} = \frac{(-2)^3 + 2(-2)^2 - 1}{5 - 3(-2)} = -\frac{1}{11}$$

Case 2. If the fraction is of the form $\frac{0}{0}$ and $\frac{\infty}{\infty}$, then, sometimes, there is HOPE!

Try to manipulate $f(x)$ by rationalizing, factoring, etc. in order to cancel. (Alternatively, L'Hôpital's Rule can be used.)

Example. Compute $\lim_{x \to 2} \frac{x - 4}{\sqrt{x} - 2}$.

Solution. The fraction has the form $\frac{0}{0}$. HOPE!

By using the difference of squares,

$$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2}=\lim_{x\to 4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-\sqrt{2}}$$
$$=\lim_{x\to 4}\sqrt{x}+\sqrt{2}=\sqrt{4}+\sqrt{2}$$
$$=2+\sqrt{2}$$

<u>Case 3.</u> If the fraction is of the form $\frac{1}{0}$, then the limit does not exist. (Note that 1 in the numerator can be any non-zero number.)

Example. Evaluate $\lim_{x\to -3}\frac{x^2-x+12}{x+3}$, if possible.

Solution. Note that $f(x)$ cannot be reduced.

The limit of the numerator is $\lim_{x\to -3}(x^2-x+12)=24$, which is not equal to zero.

The limit of the denominator is $\lim_{x\to -3}x+3=0$.

Thus, the limit does not exist.