Implicit Differentiation

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Explicit equation (i.e., y given explicitly as a function of x):

$$y = e^{3x} - x^2 + \ln x$$

Explicit equation (i.e., y given explicitly as a function of x, or x given explicitly as a function of y):

$$x^2 y + y^2 = e^{xy} + \sin(x + y)$$

For functions given IMPLICITLY:

Step 1. Assuming that y is a function of x, i.e., $y = f(x)$, take the usual derivative with respect to x, keeping in mind to use the chain rule when differentiating terms involving y (which, of course, involves multiplying by y' or $\frac{dy}{dx}$).

Step 2. Solve for y', if possible, or requested.

Example. Differentiate the following equations.

(a) $2x^3 - 3xy^2 + y^4 = -20$

(b) $xy = e^{-xy}$

Solution. (a) Step 1.

$$2x^3 - 3xy^2 + y^4 = -20$$

$$6x^2 - 3[y^2 + 2yy'x] + 4y^3 y' = 0$$

$$6x^2 - 3y^2 - 6yy'x + 4y^3 y' = 0$$

Step 2.

$$-6xyy' + 4y^3y' = -6x^2 + 3y^2$$
$$y'(-6xy + 4y^3) = -6x^2 + 3y^2 \quad ,$$
$$y' = \frac{-6x^2 + 3y^2}{-6xy + 4y^3}$$

(b) Step 1.

$$xy = e^{-xy}$$
$$y + xy' = e^{-xy}(-y - xy')$$
$$y + xy' = -e^{-xy}y - e^{-xy}xy'$$

Step 2.

$$xy' + e^{-xy}xy' = -e^{-xy}y - y$$
$$y'(x + e^{-xy}x) = -y(e^{-xy} + 1) \quad ,$$
$$y' = \frac{-y(e^{-xy} + 1)}{x(e^{-xy} + 1)} = \frac{-y}{x}$$

since $e^{-xy} + 1 \neq 0$.