Integration by Parts

Formula for Integration by Parts

\[ \int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx \]

or

\[ \int u\,dv = uv - \int v\,du \]

How to pick \( f(x) \) and \( g'(x) \), i.e., \( u \) and \( dv \)?

Here is a suggestion!

Remember the word LATE:

- L – logarithmic ex. \( \ln(x) \)
- A – algebraic ex. \( x^2 + x \)
- T – trigonometric ex. \( \sin(x) \)
- E – exponential ex. \( e^x \)

The first expression that appears in the word LATE in the integrand will be \( u \), and the rest will be \( dv \).

Take the derivative of \( u \) and integrate \( dv \). Then, use the formula for integration by parts.

Example. Find \( \int xe^{2x}\,dx \).

Solution. According to LATE, \( u = x \) and \( dv = e^{2x}\,dx \).

\[ u = x \quad \int dv = \int e^{2x}\,dx \]

\[ du = dx \quad \text{and} \quad v = \frac{e^{2x}}{2} \]
\[
\int x e^{2x} \, dx = \left( x \left( \frac{e^{2x}}{2} \right) \right) - \int \frac{e^{2x}}{2} \, dx = \frac{x e^{2x}}{2} - \frac{1}{2} \left( \frac{e^{2x}}{2} \right) + C
\]
\[
= \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C = \frac{e^{2x}}{2} \left( x - \frac{1}{2} \right) + C
\]

**Definite Integration by Parts**

The integration by parts formula can be applied to definite integrals by noting that

\[
\int_a^b u dv = uv \bigg|_a^b - \int_a^b v du.
\]

**Computing a Definite Integral by Integration by Parts**

**Step 1.** Solve the integral as an indefinite integral.

**Step 2.** Evaluate it over the interval of integration.

**Example.** Find \( \int_1^e \frac{\ln x}{x^2} \, dx \).

**Solution.**

Step 1. According to LATE,

\[
u = \ln x \quad \text{and} \quad dv = \frac{1}{x^2} \, dx = x^{-2} \, dx.
\]

\[
u = \ln x \quad \int dv = \int \frac{1}{x^2} \, dx = \int x^{-2} \, dx
\]

\[
du = \frac{1}{x} \, dx \quad \text{and} \quad v = -\frac{1}{x}
\]

\[
\int_1^e \frac{\ln x}{x^2} \, dx = \left( \ln x \left( \frac{-1}{x} \right) \right) - \left( \int \frac{-1}{x} \left( \frac{1}{x} \right) \, dx \right)
\]

\[
= -\frac{\ln x}{x} - \int x^{-2} \, dx
\]

\[
= -\frac{\ln x}{x} - \frac{1}{x} + C
\]
Step 2.

\[ \int_{1}^{e} \frac{\ln x}{x^2} \, dx = -\frac{\ln x}{x} - \frac{1}{x} \bigg|_{1}^{e} \]

\[ = \left( -\frac{\ln e}{e} - \frac{1}{e} \right) - \left( -\frac{\ln 1}{1} - \frac{1}{1} \right) \]

\[ = -\frac{2}{e} + 1 \]