A function $f(x)$ is continuous at a point $x = a$ if and only if the following conditions are satisfied:

1. $f(a)$ exists (i.e., $a$ lies in the domain of $f$)
2. $\lim_{x \to a} f(x)$ exists ($f$ has a limit as $x \to a$, i.e., the limit is a real number)
3. $\lim_{x \to a} f(x) = f(a)$ (the limit equals the function value)

If one or more of these conditions fail, then $f(x)$ is not continuous at a point $x = a$.

Example. Consider the function $f(x) = \begin{cases} \frac{x^3 + 2x^2 - 8x}{x^2 - 4}, & \text{if } x \neq 2 \\ 4, & \text{if } x = 2 \end{cases}$.

(a) Is $f(x)$ continuous at $x = -2$?
(b) Does $\lim_{x \to 2} f(x)$ exist?
(c) Is $f(x)$ continuous at $x = 2$?

Solution. (a) $f(-2) = \frac{(-2)^3 + 2(-2)^2 - 8(-2)}{(-2)^2 - 4} = \frac{16}{0}$.
Thus, $f(-2)$ is not defined, and hence, $f(x)$ is NOT continuous at $x = -2$.

(b) Note that one-side is not necessary, but for completeness, it is shown.
\[
\lim_{x \to 2^{-}} f(x) = \lim_{x \to 2^{-}} \frac{x^3 + 2x^2 - 8x}{x^2 - 4} = \lim_{x \to 2^{-}} \frac{x(x + 4)(x - 2)}{(x + 2)(x - 2)} = \lim_{x \to 2^-} \frac{x(x + 4)}{x + 2} = \frac{2(2 + 4)}{2 + 2} = 3
\]

\[
\lim_{x \to 2^{+}} f(x) = \lim_{x \to 2^{+}} \frac{x(x + 4)}{x + 2} = \frac{2(2 + 4)}{2 + 2} = 3
\]

Yes, the limit exists, that is, \( \lim_{x \to 2} f(x) = 3 \).

(c) Note that \( f(2) = 4 \neq 3 = \lim_{x \to 2} f(x) \).

By the definition of continuity, \( f(x) \) is NOT continuous at \( x = 2 \).