VARIGRAM ANALYSIS OF THE SPATIAL GENETIC STRUCTURE OF CONTINUOUS POPULATIONS USING MULTILOCUS MICROSATTELITE DATA

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Manuscript received September 7, 2004
Accepted for publication December 2, 2004

ABSTRACT

A geostatistical perspective on spatial genetic structure may explain methodological issues of quantifying spatial genetic structure and suggest new approaches to addressing them. We use a variogram approach to (i) derive a spatial partitioning of molecular variance, gene diversity, and genotypic diversity for microsatellite data under the infinite allele model (IAM) and the stepwise mutation model (SMM), (ii) develop a weighting of sampling units to reflect ploidy levels or multiple sampling of genets, and (iii) show how variograms summarize the spatial genetic structure within a population under isolation-by-distance. The methods are illustrated with data from a population of the epiphytic lichen Lobaria pulmonaria, using six microsatellite markers. Variogram-based analysis not only avoids bias due to the underestimation of population variance in the presence of spatial autocorrelation, but also provides estimates of population genetic diversity and the degree and extent of spatial genetic structure accounting for autocorrelation.

METHODS for the analysis of spatial genetic structure have mostly been developed for single-locus, diploid genotypic data such as provided by isozymes (Smouse and Peakall 1999). In contrast to this latter marker type, microsatellite data also contain information on repeat numbers of individual gene copies. Microsatellite markers are often highly variable, and differences in allele size are interpreted in the light of alternative evolutionary models. Under the infinite allele model (IAM), any mutation is assumed to lead to a new allele, whereas under the stepwise mutation model (SMM), mutation increases or decreases the number of repeats at a microsatellite locus most likely by one (Balloux and Goudet 2002). Neither of these two extreme mutation models seems to fit perfectly to microsatellite loci, so that measures based on IAM and SMM are often reported together (Balloux and Lugon-Moulin 2002). The difference between statistical measures (see below) under the two models is assumed to indicate the relative importance of mutation and drift (Hardy 2003). Population genetic analyses are based on gene diversity under the IAM (cf. $F_{ST}$) and on molecular variance under the SMM (cf. $R_{ST}$). $F_{ST}$ and $R_{ST}$ quantify the differentiation of isolated populations assuming random mating within and restricted gene flow among populations. Both $F_{ST}$ and $R_{ST}$ can be adapted to pairwise comparisons, and Mantel tests are used to test the correlation with geographic distance between pairs of populations (Hardy and Vekemans 2002). However, limited gene movement can cause isolation-by-distance effects even within continuous populations. The resulting spatial genetic structure within a population can be summarized by kinship for IAM (Loiselle et al. 1995) or relationship coefficients for SMM (Streiff et al. 1998). Kinship and relationship coefficients assess the similarity of homologous alleles between individuals and may be expressed as a function of geographic distance. Statistical tests for isolation-by-distance within continuous populations often involve either a Mantel permutation test of Moran’s I (or related correlation coefficients, e.g., Smouse and Peakall 1999) or join-count statistics (Epperson 2003).

When assessing genetic diversity, it may be necessary to exclude comparisons of gene copies within individuals if they cannot be assumed to be independent. For organisms with variable ploidy levels within populations such as Taraxacum sp. (Meirmans et al. 2003; van der Hulst et al. 2003), individuals with a high ploidy level will receive more weight in the estimation of the population genetic diversity than, e.g., diploid individuals unless ploidy level is accounted for. A similar problem arise for clonal organisms, where the multiple sampling of ramets from the same genetic individuum (genet) can bias any measure of genetic structure of a population (Parks and Werth 1993; Balloux et al. 2003; Hammerli and Reusch 2003). This is commonly taken into account by retaining a single sample per genet, either assuming the center of a clonal patch to be its origin or randomly selecting one sample per genet (Reusch et al. 1999; Chung and Epperson 2000; Hammerli and Reusch 2003). Both approaches may, however, lead to
a considerable loss of information and increased error in the description of the spatial genetic structure within populations.

Vekemans and Hardy (2004) identified some important problems and common misuses of spatial analysis in population genetics.

i. The spatial genetic structure is often described in terms of a maximum distance to which such structure extends. The common practice of assessing the extent of spatial genetic structure by the distance at which a Moran’s I correlogram reaches zero (e.g., Epperson 2003) is misleading, as this estimate depends strongly on the sampling design (Vekemans and Hardy 2004).

ii. The presence of nonrandom spatial genetic structure can be tested using Mantel permutation tests for a series of distance classes, and a Bonferroni correction is applied to account for multiple tests. Vekemans and Hardy (2004) caution that while the uncorrected test is too liberal, the correction makes it too conservative, and they argue that this approach should not be used to determine the scales of spatial genetic structure, as the null hypothesis is only the overall absence of spatial genetic structure.

iii. The amount of spatial genetic structure should not be assessed from the value (e.g., of Moran’s I) for the first distance class, as this absolute value depends strongly on the sampling design (Fenster et al. 2003; Vekemans and Hardy 2004).

iv. Estimating biological parameters, such as dispersal distances, is valid only if the observed spatial genetic structure represents a true isolation-by-distance pattern at dispersal-drift equilibrium (Vekemans and Hardy 2004), thus assuming that the patterning results only from limited dispersal, that it has reached a stationary phase, and that the scale of the study is appropriate (Vekemans and Hardy 2004).

Moran’s I, Mantel tests, and join-count statistics were borrowed from the general field of spatial statistics, originally developed, e.g., in geography, and adapted to population genetic data and questions as necessary. Other measures of spatial genetic structure, such as kinship or relationship coefficients, were developed specifically for genetic data and are little integrated with spatial statistical theory. However, many of the above problems are of a general nature and not specific to population genetics. Particularly, variogram modeling as developed in geostatistics may provide explanations and alternatives for the problems raised by Vekemans and Hardy (2004). The term variogram refers to a plot of the semivariance (see below) against distance. The well-known Geary’s c correlogram is actually a standardized variogram (Legendre and Legendre 1998). Several population genetic measures and methods rely on the semivariance, namely the genetic distance measure by Goldstein et al. (1999) and the $R_{ST}$ statistic (Slatkin 1995). Nonetheless, variogram modeling is rare in population genetics. Piazza and Menozzi (1983) proposed a variogram of differences in allele frequencies between populations, and Monestiez and Goulard (1997) provided an application of multivariate geostatistical analysis to genetic data, but neither approach found much resonance in the population genetic literature.

Wagner (2003, 2004) developed a formal integration of multivariate analysis and geostatistics in the context of plant community ecology. The crucial point of such an integration of spatial and nonspatial analysis is that the semivariance partitions the estimate of the population variance by distance class (Wagner 2003). Hence, the semivariance can be used to partition the results of nonspatial analyses, such as population estimates of genetic diversity, by distance (multiscale ordination), and variograms can be interpreted in an ecologically more meaningful way.

This article extends the spatial partitioning of variance to population genetic data and problems. A GEOSTATISTICAL PERSPECTIVE introduces key geostatistical concepts and methods and discusses the sensitivity of commonly used measures of autocorrelation and population variance. Development of methods pursues three specific objectives: (i) to derive a spatial partitioning of measures of genetic diversity compatible with IAM and SMM, (ii) to develop a method for weighting sampling units to reflect different ploidy levels or multiple sampling of ramets within genets without data reduction, and (iii) to show how variogram modeling can be used for estimating population genetic parameters and summarizing the spatial genetic structure within populations. The methods are illustrated with a worked example (Appendix) and with an application to empirical microsatellite data from a population of the haploid, tree-colonizing (epiphytic) lichen Lobaria pulmonaria. We conclude with considerations for the robust estimation of the spatial genetic structure of continuous populations.

A GEOSTATISTICAL PERSPECTIVE

Geostatistical concepts and methods: Spatial autocorrelation and stationarity: Spatial autocorrelation refers to the common phenomenon that nearby observations tend to be more similar than distant ones. Positive spatial autocorrelation is assumed to result from any kind of spatial process, such as pollen flow or seed dispersal in plants. When a variable is studied in space, the observed spatial autocorrelation can be quantified for various purposes (Fortin et al. 2001), such as: (i) testing for the presence of autocorrelation, e.g., to meet assumptions for estimating population characteristics; (ii) assessing the range of autocorrelation, i.e., the distance beyond which observations are spatially independent; (iii) fitting...
Specifically, it is common to assume weak stationarity, distance falling into distance class correlation must be the same throughout the study area. Geary’s populations. However, geostatistical analysis requires some process, such as dispersal distances and differences among a pair of samples belongs to distance class structure; and (iv) inferring about the underlying spatial pair of observations can contribute to the estimates for larger distances. It is therefore customary to limit the description of the spatial structure to half the maximum distance between sampling units (Burrough 1995).

Correlograms and the empirical variogram: Geostatistics considers four statistical moments of a random variable: (i) its mean, (ii) variance, (iii) covariance, and (iv) semivariance (Burrough 1995). Spatial autocorrelation can be quantified on the basis of covariance (Moran’s I) or semivariance (empirical variogram and Geary’s c correlogram). Bertorelle and Barbujani (1995) derived alternative versions of Moran’s I and Geary’s c for binary-coded data on the basis of, e.g., DNA sequences or RFLP patterns. Correlograms are standardized through division by the sample variance (Moran’s I) or population variance (Geary’s c; CLIFF and Ord 1981):

Moran’s I:

\[ I(r) = \frac{\text{covariance}}{\text{variance}} = \frac{(1/n) \sum_{a \neq b} x_a \bar{x}_b (y_a - \bar{y})(y_b - \bar{y})}{(1/N) \sum (y_a - \bar{y})^2}. \]

Geary’s c:

\[ c(r) = \frac{\text{semivariance}}{\text{variance}} = \frac{(1/2n) \sum_{a \neq b} (y_a - y_b)^2}{(1/(N-1)) \sum (y_a - \bar{y})^2}. \]

Empirical variogram:

\[ \gamma(r) = \text{semivariance} = \frac{1}{2n} \sum_{a \neq b} (y_a - y_b). \]

Given N samples, these coefficients are calculated on the basis of pairs of samples a and b falling into a series of distance classes r. The Kronecker weight \( x_a^{(r)} \) for the pair of observations a and b takes the value \( x_a^{(r)} = 1 \) if a pair of samples belongs to distance class r and \( x_a^{(r)} = 0 \) otherwise, and n, is the sum of the weights \( x_a^{(r)} \) for the given distance class, i.e., the number of pairs of gene copies a and b from two samples separated by a distance falling into distance class r. However, n, decreases for large distance classes r, and bias may arise from the fact that only the observations from the edge of the sampled population can contribute to the estimates for larger distances. It is therefore customary to limit the description of the spatial structure to half the maximum distance between sampling units (Cressie 1993).

Figure 1A shows the empirical variogram and Figure 1B shows Moran’s I and Geary’s c correlograms of an artificial, spatially autocorrelated random variable. Geary’s c correlogram is a rescaled version of the empirical variogram, and Moran’s I correlogram resembles, but is not identical to, \( 1 - c(r) \) (Legendre and Legendre 1998). In the absence of spatial autocorrelation, the expected value of Geary’s c is \( E[c(r)] = 1 \), whereas the expected value of Moran’s I is \( E[I(r)] = -1/(N-1) \), which approaches zero for large sample sizes N (Sokal and Wartenberg 1983; Epperson 2003).

Variogram modeling: The autocorrelation structure can be modeled by fitting a theoretical variogram model to the empirical variogram. The elementary theoretical variograms suitable for modeling patterns due to a single, stationary spatial process are defined by the following parameters: (i) model family, such as exponential, spherical, or Gaussian; (ii) nugget variance, i.e., the variance among adjacent samples; (iii) range, or the distance beyond which observations are spatially independent; and (iv) sill, the constant variance among spatially uncorrelated samples (Figure 1A; Isaaks and Srivastava 1989).

Figure 1.—Empirical variogram (A) and correlograms (B) for an artificial, spatially autocorrelated random variable simulated on a grid of 30 × 30 cells. Each symbol denotes the semivariance \( \gamma(r) \) (circles), Geary’s c(r) (squares), or Moran’s I(r) (triangles) calculated from all pairs of samples falling into each distance class r. The value for the last distance class of each series contains all pairs separated by >20 units and is drawn at the mean of the respective distances. (A) The solid line represents the fitted exponential variogram model. The dotted line (sill) indicates the population variance as estimated accounting for autocorrelation. The dashed line indicates the practical range, where the curve reaches 95% of the sill. The intercept (nugget variance) is the variance component that is not spatially structured. (B) The solid line indicates the expected value of Moran’s I(r), which is very close to zero, whereas the dashed line marks the expected value of Geary’s c(r), which equals one.
The autocorrelated data were generated by dividing an ordered vector of 500 random values from a standard normal distribution into groups of five consecutive values. The entire groups and the values within each group were reordered at random, representing a hypothetical transect where always five neighboring locations would show very similar values, with random steps between groups. The data set has an expected variance of 1.0 and the proportion of (autocorrelated) within-group comparisons is 0.016. Three types of samples were taken from the transect: (i) a random sample from all locations, (ii) a systematic sample selecting every fifth location, and (iii) a clustered sample, where entire groups of five neighboring locations were selected at random. Data simulation and sampling were repeated 1000 times for each sample size of 10, 20, 50, or 100.

The variogram can be interpreted as a distance-dependent estimate of the population variance (WAGNER 2003). The commonly used “unbiased” estimator of population variance, $\bar{V} = \frac{1}{N-1} \sum (y_i - \bar{y})^2$, $(N - 1)$, assumes independent, spatially uncorrelated observations, which would correspond to a strictly horizontal empirical variogram. In essence, this requires the assumption of a panmictic population with random dispersal, which is likely to be violated in most natural systems.

Spatial autocorrelation reduces the variance between closely spaced pairs of observations. The following simulation illustrates the consequences of spatial autocorrelation for estimating the population variance and thus for rescaling Moran’s $I$ and Geary’s $c$ correlograms. An artificial, autocorrelated variable was sampled in different ways and the estimates of the population variance, averaged over many replicate simulations, were compared to the true value. We compared three sampling strategies: (i) systematic sampling, with a spacing known to be larger than the range of spatial autocorrelation to obtain spatially uncorrelated, independent sampling units; (ii) random sampling; and (iii) stratified or clustered sampling, selecting groups of nearby locations to obtain an appropriate representation of short distance classes for spatial analysis. The criteria for comparison were accuracy, i.e., the absence of bias so that the mean of all replicate estimates is close to the true population value, and precision, i.e., low variability of replicate estimates (PALMER 1990). For details of the simulation experiment, see the Table 1 legend.

The systematic and the random samples provided unbiased estimates, independent of sample size (Table 1). Precision increased with sample size; i.e., the standard deviation of the estimates was reduced. For small sample sizes, where the chances of randomly selecting autocorrelated samples were small, the systematic and the random samples reached a similar precision. With
increasing sample size, however, the random samples provided a lower precision than the systematic samples. This effect was due to the increasing number of comparisons between autocorrelated samples, not their proportion. Parametric statistical tests assume spatially uncorrelated samples, which in this simulation corresponds to the systematic sampling design. For a spatially autocorrelated variable, the increased variability of estimates from a random sample may render such tests too liberal. This means that the actual probability of rejecting the null hypothesis when it is true (type I error) may be larger than the stated significance level $\alpha$.

On average, the clustered samples strongly underestimated the population variance (Table 1). This negative bias was reduced with increasing sample size, as more and more clusters of samples were selected, thus reducing the proportion of comparisons between autocorrelated pairs of samples. In fact, the variance of the estimates based on clustered samples was comparable to the variance for systematic samples with a five times smaller sample size, which can be explained by the sampling of clusters of five strongly autocorrelated locations. However, the clustered samples provided biased estimates, whereas the corresponding systematic samples were unbiased. Hence, the unbiased variance estimator may be negatively biased due to spatial autocorrelation. The magnitude of this systematic bias will depend on the spatial autocorrelation structure and the proportion of autocorrelated samples, which are functions of the spatial configuration of the sample rather than sample size. One may argue that the spatial autocorrelation structure is an inherent characteristic of a population. However, because the estimate of the population variance depends on the sampling design, it should be based on independent samples.

Correlograms imply division by the sample or population variance (see Equations 1 and 2). Because of this, it follows that (i) the actual values of Moran’s $I(r)$ and Geary’s $c(r)$ depend on the spatial configuration of the sample, and (ii), for a stationary process, $I(r)$ reaches a value slightly below zero, and $c(r)$ a value above one, for distances beyond the range of spatial autocorrelation. The exact deviation cannot be predicted without knowing the spatial autocorrelation structure and the details of the sampling design. This is not accounted for by subtracting $E[I(r)] = -1/(N - 1)$ for Moran’s $I(r)$. On the other hand, an empirical variogram can be used to estimate the real population variance accounting for autocorrelation, usually by fitting a theoretical variogram model. Hence, the above exemplified problem of Moran’s $I$ and Geary’s $c$ can be avoided.

**DEVELOPMENT OF METHODS**

**Definition of genetic variograms:** This paragraph shows how variance-based measures of genetic distance can be estimated from pairwise comparisons, so that variograms can be defined that provide an estimate of genetic diversity as a function of geographic distance.

**Variogram of molecular variance:** The univariate definition of a variogram (Equation 3) can be extended to multivariate data. Thus, under the SMM, $y_i$ and $y_j$ are not two observations of the allele size $y_i$ of a single locus $l$, but vectors $Y_i$ and $Y_j$ of two observations of the number of repeats at $L$ loci. The empirical semivariance $\hat{\gamma}(r)$ becomes half the squared Euclidean distance between $Y_i$ and $Y_j$ and is equal to the sum of the empirical semivariances $\gamma_i(r)$ of the number of repeats $y_i$ (Wagner 2003):

$$\hat{\gamma}(r) = \frac{1}{2n_{r < i<j}^\ell} \sum_{i<j=1}^L \sum_{r} y_{ij}^2 (y_i - y_j)^2$$

$$= \frac{1}{n_{r < i<j}^\ell} \sum_{i<j=1}^L \sum_{r} y_{ij}^2 (y_i - y_j) = \sum_{r} \gamma_i(r).$$  

(4)

More generally, a multivariate variogram can be defined as a weighted average $\mathbb{V}(r)$ of the component variograms $\gamma_i(r)$, with Equation 4 as the special case of $w_l = 1$:

$$\mathbb{V}(r) = \sum_{l} w_l \gamma_l(r).$$  

(5)

Most often, the variograms of the $L$ loci will be weighted by $w_l = 1/L$.

Under SMM, genetic diversity is related to differences in allele size, where allele size $y_i$ is defined as the number of repeats of gene copy $a$ at locus $l$. The molecular variance of a single locus $l$ with $k$ alleles can be defined as

$$\hat{\gamma}_l = \frac{N}{N - 1} \sum_{a} p_a (y_a - \bar{y})^2 = \frac{1}{N - 1} \sum_{a} (y_a - \bar{y})^2$$

(Renwick et al. 2001), where

$$\bar{y} = \frac{1}{N} \sum_{a} y_a.$$

(6)

Equations 4 and 6 provide a distance-dependent estimate $\hat{\mathbb{V}}(r)$ of the molecular variance $\hat{\gamma}$, averaged over $L$ loci, which can be used as a within-population analog to $R_{st}$ to investigate isolation-by-distance effects within a continuous population:

$$\hat{\mathbb{V}}(r) = \sum_{l} w_l \hat{\gamma}_l(r).$$  

(7)

The statistical significance of a departure of $\hat{\mathbb{V}}(r)$ from its expected value under the null hypothesis of no spatial autocorrelation can be tested in a Mantel permutation test (Legendre and Legendre 1998). If the alternative hypothesis is positive spatial autocorrelation at short distances, a one-sided test with a progressive Bonferroni correction can be applied, where the significance level for the $k$th distance class is $\alpha/k$ (Hewitt et al. 1997; Legendre and Legendre 1998; Lichstein et al. 2002). This correction is appropriate when significant autocorrelation is hypothesized to occur in the smallest distance classes and the aim is to determine the extent of spatial structure (Legendre and Legendre 1998).
Gene diversity is a key parameter in population genetics under IAM. Gene diversity $H_i$ is the probability that two gene copies sampled with replacement differ at locus $l$. The unbiased estimator of gene diversity, $\hat{H}_l$, at locus $l$ for a sample of $N$ gene copies of $k$ different alleles is

$$\hat{H}_l = \frac{N}{N-1} \left(1 - \sum \hat{p}_i^2\right)$$

(NEI 1978).

On the basis of Equations 8 and 9, the variogram of multilocus gene diversity $\hat{H}$ can be defined as

$$\hat{H}(r) = \sum_{l=1}^{L} w_l \hat{\gamma}_l(r) = \sum_{l=1}^{L} \sum_{a<b}^{l} \sum_{i=1}^{l} \frac{x_{ab}^{(i)}}{2n_l} (z_{iab} - z_{iab})^2.$$

(10)

$\hat{H}(r)$ provides a within-population analog to $F_{ST}$. As with $\hat{V}(r)$, the significance of an observed autocorrelation in $\hat{H}(r)$ can be tested with a Mantel permutation test.

**Variogram of genotypic diversity:** Genotypic diversity measured by Simpson’s diversity $D$ is similar to single-locus gene diversity $H_l$, but, instead of allele $k$, the multilocus genotype $g$ is used, so that $D$ is the probability of sampling two individuals of different multilocus genotypes. The unbiased estimator of genotypic diversity is

$$\hat{D} = \frac{N}{N-1} \left(1 - \sum \hat{p}_i^2\right).$$

(11)

The variogram of genotypic diversity $\hat{D}$ (Simpson diversity) is obtained by coding each multilocus genotype by a dummy variable $z_g$, which takes the value $z_g = 1$ if individual $i$ is of genotype $g$ and $z_g = 0$ if it is not. For a haploid organism, the analysis is based on gene copies, whereas for diploid organisms, genotype coding would normally reflect diploid genotypes. The variogram of genotypic diversity, $\hat{D}(r)$, estimates the probability of sampling two individuals of different multilocus genotypes as a function of their distance in space and is calculated as

$$\hat{D}(r) = \sum_{a<b} \sum_{i=1}^{l} \frac{x_{ab}^{(i)}}{2n_l} (z_{iab} - z_{iab})^2.$$

(12)

Accounting for ploidy levels and clonality: This paragraph introduces a weighting scheme that accommo-
distance of the organism, then dispersal parameters can be estimated from $Sp$ (Veremans and Hardy 2004).

**Exponential variogram model:** Assuming an exponential relationship in a two-dimensional case (see above), the spatial genetic structure of a population can be summarized by fitting an exponential variogram model (Figure 1),

$$\gamma(r) = C_0 + C_1[1 - e^{-b(r/b)}]$$

(14)

where $C_0$ is the nugget variance, or the proportion of the variance that is not spatially structured, and $C_1$ is the spatially structured variance component (Legendre and Legendre 1998). The sill $C = C_0 + C_1$ provides an estimate of the population variance based on spatially independent samples, i.e., accounting for spatial autocorrelation. The relative size of the nugget provides an estimate of $F_N$: $1 - C_0/C = C_1/C = \hat{F}_N$. This can be set to $F_0$ by fitting a fixed-nugget model, constraining the nugget variance to the observed semivariance for the first distance class.

The exponential model approaches the sill $C$ asymptotically. Therefore, the range or slope parameter $b$ indicates the practical range of the exponential variogram, plied Biosystems, Foster City, CA), followed

that

sill (i.e, the distance at which the curve reaches 95% of the

Journel and Huijbregts 1978). It can be shown that $b = -3/\theta$. Directional dispersal or migration may lead to anisotropy, where the genetic structure depends on direction. If there is reason to expect anisotropy, directional variograms can be fitted, providing estimates of the slope parameter $b$ for different compass directions.

A confidence interval for the slope parameter $b$ may be estimated using the permutation method for the confidence interval for the matrix regression coefficient proposed by Manly (1997). The residuals of the exponential model are randomly permuted many times to obtain the reference distribution of the correlation of the residuals with distance. A series of exponential models with varying range parameters is derived, and the confidence intervals are determined at which the correlation of the residuals of these new models with distance is as strong as for the $\alpha/2$ and the $(1 - \alpha)/2$ quantiles of the reference distribution. The two critical values provide the lower and upper limits of the confidence interval for the range parameter $b$.

**APPLICATION TO THE GENETIC STRUCTURE OF L. PULMONARIA**

**Model organism:** *L. pulmonaria* is a foliose epiphytic lichen species of humid temperate and boreal regions of the northern hemisphere and cooler parts of the tropics (Yoshimura 1971). This clonal and recombinant species (Walser et al. 2004), which produces both vegetative and sexual diaspores, is considered endangered in most parts of Central Europe (Wirth et al. 1996) and in other industrialized regions. It is used as an indicator of ecological continuity (Rose 1992) in natural forests and traditionally managed agro-forestry landscapes, such as wooded pastures and chestnut orchards (Scheidegger et al. 2002).

**Data:** We studied the spatial genetic structure of a continuous population of *L. pulmonaria* from the Swiss Jura Mountains. A hierarchical random sample of 461 thalli was collected from a pasture-woodland landscape. In a first step, 100 circular plots of 1 ha were randomly selected from the wooded parts of the study area. Within each plot, all suitable trees exceeding 5 cm in diameter at breast height were searched for *L. pulmonaria*. A maximum of 24 thalli were randomly selected from different trees in each of the 24 plots where the lichen was present. If there were <24 colonized trees, multiple thalli were sampled from the same tree, and if there were <24 thalli in a plot, every thallus found was included. This results in a heterogeneous data set that could exhibit spatial autocorrelation at varying scales.

DNA extraction and fragment length determination at six microsatellite loci (LPu03, LPu09, LPu15, LPu16a, LPu20a, and LPu27a), specific to the haploid mycobiont, using an ABI 3100-Avant automated sequencer (Applied Biosystems, Foster City, CA), followed Walser et al. (2003). Allele assignment was performed using GENEPUTER 2.5 software (Applied Biosystems).

**Statistical analyses:** Omni-directional variograms of molecular variance $V(r)$ and gene diversity $H(r)$ were calculated according to Equations 7 and 10, giving equal weight to each of the six loci. The first distance class of $r = 0$ contained pairs of thalli from the same tree. The lag distance was 50 m. The last distance class contained all sample comparisons at distances $>450$ m. Autocorrelation was tested per distance class using a one-sided Mantel test with 500 permutations of the thalli and a progressive Bonferroni correction of $\alpha = 0.05/k$ for the $k$th distance class up to the first nonsignificant value.

A second set of variograms, $V'(r)$ and $H'(r)$, was calculated weighted each thallus by the number of occurrences of its multilocus genotype within the population, using modified weights $x^2_{ij}^2$ (Equation 13). Autocorrelation was again tested per distance class using a one-sided Mantel test with 500 permutations of the multilocus genotypes, using the same settings as above.

An isotropic and four-directional variograms of genotypic diversity $D(r)$ were also calculated according to Equation 12, giving equal weight to all samples. Exponential variogram models were fitted to all variograms, using the weighted least-squares algorithm (Cressie 1993) that minimizes the expression

$$\sum_r n_r \left( \frac{\bar{\gamma}(r; C_0, C_1, b)}{\gamma(r; C_0, C_1, b)} \right)^2,$$

(15)

where $\gamma(r; C_0, C_1, b)$ is the fitted semivariance for distance class $r$ on the basis of the exponential model with parameters $C_0$, $C_1$, and $b$.

All calculations were performed in R (Ihaka and Gentleman 1996). The exponential variograms were fitted
Results and discussion: On the basis of the six microsatellite markers, we found 92 multilocus genotypes of the haploid mycobiont of Lobaria pulmonaria. All but 9 multilocus genotypes occurred in single 1-ha plots, and only 1 was spread over >210 m. The probability of origin by recombination was <0.003 for all recurrent multilocus genotypes, suggesting that they arose from clonal propagation. Weighting for recurrent genotypes drastically reduced effective sample size from 461 thalli to 92 multilocus genotypes. For recurrent genotypes, pairwise comparisons were distributed over several distance classes: the first distance class contained an equivalent of 36.8 pairs (instead of 1588 for all samples) and the other distance classes up to 450 m comprised 47.6–148.6 pairs (instead of >2000).

The spatial genetic structure of the studied Lobaria pulmonaria population consisted of two patterns caused by clonal reproduction [variogram of genotypic diversity, \( D(r) \)] and sexual reproduction [variograms of molecular variance, \( V(r) \), and gene diversity, \( H(r) \); Figure 2]. Weighting for recurrent genotypes reduced the autocorrelation of the first distance class and the range estimate both for molecular variance and for gene diversity (Table 2). After weighting for clones, the range parameters \( b \) of the fitted exponential variogram models were smaller for molecular variance, \( V(r) \), and gene diversity, \( H(r) \), than for genotypic diversity, \( D(r) \), suggesting larger dispersal distances or several steps of clonal dispersal (Table 2). On the other hand, genotypic diversity showed a higher degree of autocorrelation for the first distance class, \( F_{xy} \), which consisted of pairs of samples from the same tree. The fitting of directional variograms for genotypic diversity revealed that spatial genetic structure extended further in the main wind direction (WSW–ENE; Vittoz 1998) than in the other directions (Table 2).

The conventional estimates of the population variance, \( \hat{V} \), \( \hat{H} \), and \( \hat{D} \), slightly underestimated the variance for spatially independent samples, i.e., the total sill \( C \) for all three measures of diversity (Table 2). In this specific example, however, weighting for recurrent genotypes largely compensated this bias.

**DISCUSSION**

**Advantages of variogram analysis:** A geostatistical perspective on spatial genetic structure can provide explanations to many of the issues raised by Vekemans and Hardy (2004) and suggest new approaches to address them. First, the sampling design has a strong influence on the absolute values of Moran’s \( I \) or other coefficients of relatedness, and this may severely limit comparability between studies. The sampling design also affects the distance at which these measures reach their expected value in the absence of spatial structuring. Therefore, this distance provides only a somewhat arbitrary estimate of the extent of spatial genetic structure (Vekemans and Hardy 2004). This problem affects the analysis of kinship structure with Moran’s \( I \) or relationship coefficients, where empirical values for larger distances tend to be slightly below zero, whereas in theory negative kinship coefficients are not allowed (Barbujani 1987).

Our simulation experiment showed that this effect is not simply due to sample size, but relates to the inclusion of autocorrelated samples in the estimation of the population variance, which is commonly used as a reference for rescaling correlograms and other measures of relatedness. Variogram modeling, on the other hand, provides an estimate of the population variance accounting for spatial autocorrelation, and its model parameters are scaled by this corrected estimate.

Second, Vekemans and Hardy (2004) suggested that if a plot of \( F(r) \) against distance \( r \), e.g., a Moran’s \( I \) correlogram, decreases steadily until some distance \( x \) and shows no further trend, this distance may be interpreted as the extent of spatial genetic structure. In geostatistical terms, this means that if the variogram represents a stationary spatial process as indicated by the presence of a sill, the range \( b \) can be estimated. Rather than visually identifying a critical distance at which the sill is reached, one would fit an exponential variogram model and estimate the practical range, where the curve reaches 95% of the sill. This provides an estimate of the extent of spatial genetic structure. A confidence interval for the range parameter can be constructed using a
TABLE 2

<table>
<thead>
<tr>
<th>Diversity measure</th>
<th>Weighting</th>
<th>Variogram modeling</th>
<th>Conventional estimation:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( \hat{F}_N = \frac{C_1}{C_0 + C_1} )</td>
<td>( \hat{V}, \hat{H}, \hat{D} )</td>
</tr>
<tr>
<td>Molecular variance</td>
<td>All samples</td>
<td>0.93</td>
<td>121.5</td>
</tr>
<tr>
<td></td>
<td>Weighted for clones</td>
<td>0.66</td>
<td>58.2</td>
</tr>
<tr>
<td>Gene diversity</td>
<td>All samples</td>
<td>0.87</td>
<td>135.9</td>
</tr>
<tr>
<td></td>
<td>Weighted for clones</td>
<td>0.55</td>
<td>69.3</td>
</tr>
<tr>
<td>Genotypic diversity</td>
<td>All samples</td>
<td>0.71</td>
<td>106.5</td>
</tr>
<tr>
<td></td>
<td>WSW–ENE</td>
<td>—</td>
<td>158.7</td>
</tr>
<tr>
<td></td>
<td>WNW–ESE</td>
<td>—</td>
<td>77.7</td>
</tr>
<tr>
<td></td>
<td>SSW–NNE</td>
<td>—</td>
<td>89.7</td>
</tr>
<tr>
<td></td>
<td>NNW–SSE</td>
<td>—</td>
<td>110.7</td>
</tr>
</tbody>
</table>

Estimated parameters of the exponential variogram model were fitted to the variograms of molecular variance and gene diversity for a population of *Lobaria pulmonaria* (\( N = 461 \)) assessed with six microsatellite markers, with and without accounting for recurrent genotypes, and for genotypic diversity (clonal structure), with and without accounting for compass direction. The relatedness between immediate neighbors, \( F_N \), is estimated from the autocorrelation of the first distance class of thalli taken from the same tree. The range parameter \( b \) denotes the distance at which the curve reaches 95% of the sill and provides an estimate of the extent of spatial genetic structure. The total sill \( C \) estimates the population diversity (molecular variance, gene diversity, or genotypic diversity) accounting for spatial autocorrelation, whereas the conventional estimators \( \hat{V}, \hat{H}, \hat{D} \) do not account for autocorrelation and are, therefore, susceptible to bias.

Method developed for matrix regression coefficients by Manly (1997).

Third, the interpretation of Moran’s I as a correlation coefficient or of other measures as the absolute degree of kinship or relationship is jeopardized by the dependence of the empirical values on the sampling design through an implicit rescaling (see above). The proposed empirical variograms, however, have direct interpretations independent of the sampling design, as they provide distance-dependent estimates of molecular variance \( \hat{V} \), gene diversity \( \hat{H} \), and genotypic diversity \( \hat{D} \), providing within-population analogs to the population pairwise \( R_{ST} \) and \( F_{ST} \) statistics.

The variogram of molecular variance, \( \hat{V}(r) \), for microsatellite data has a straightforward interpretation as the variance in the number of repeats expected for samples at a given distance in geographic space. It corresponds directly to a plot of (half) the sum of squared size differences as used in AMOVA of microsatellite data (Schneider et al. 2000) or of \( D_0/2 \), where \( D_0 \) is the average squared difference in repeat numbers for two alleles drawn from the same population (Goldstein et al. 1995). The variogram of genotypic diversity, \( \hat{D}(r) \), can be interpreted as the probability of sampling two different multilocus genotypes as a function of their spatial distance. The interpretation of the variogram of gene diversity, \( \hat{H}(r) \), is the probability of sampling two different alleles given their distance in geographic space, averaged over different loci. For a single locus, it corresponds exactly to a plot of the proportion of unlike links against distance, but the variogram definition is computationally simpler than the explicit coding of links. This method is related to the \( cc \)-correlogram proposed by Bertorelle and Barbujani (1995), which involves division by the population variance, and could easily be adapted to dominant markers such as randomly amplified polymorphic DNA (RAPD), intersimple sequence repeat (ISSR), or amplified fragment length polymorphism (AFLP) markers.

Fourth, Vekemans and Hardy (2004) proposed a new statistic for quantifying spatial genetic structure, \( S_p = -b/\hat{F}_N \), which arguably is more robust than either of the two component measures that are sensitive to the sampling design (see above). Rather than taking the ratio of two potentially biased quantities, variogram modeling accounts for the source of this potential bias by estimating the variance between uncorrelated samples. Hence, the variogram parameters can be directly compared between studies. Furthermore, \( F_N \) can be estimated in two different ways. If the first distance class contains the direct neighboring samples, thus representing the smallest possible distance, the semivariance for this distance class can be used as an estimate of \( F_N \) (fixed nugget-effect model). Alternatively, if the first distance class also contains not directly adjacent samples, the nugget effect needs to be fitted, providing an estimate of \( F_N \).

**Robust estimation of spatial genetic structure:** Weighting for clonality and ploidy levels. The lichen example illustrated the importance of distinguishing between spatial patterns of clonality and of genetic diversity resulting from sexual recombination. Specifically, it is crucial to account for clonal patterns when analyzing patterns of genetic diversity within a population. The confounded pattern does...
not represent the average of the two component patterns, but their multiplication, so that the degree and extent of spatial genetic structure may be severely over-estimated.

For diploid organisms, the weighting results in measures similar to the kinship coefficient by Loisellet et al. (1995) and the relationship coefficient of Streiff et al. (1998), as links within individuals are excluded. The weighting proposed here is more general and can equally be used for organisms with variable ploidy levels and applied to the correlation coefficient \( r \) by Smouse and Peakall (1999) or to join-count statistics (Epperson 2003). The proposed weighting of clones solves the problem of arbitrary resampling of recurrent genotypes, which may bias the analysis of spatial genetic structure within continuous populations (Reusch et al. 1999; Hämmerli and Reusch 2003). Whether to weight for recurrent genotypes or not will depend on the research question (e.g., dispersal distances vs. distances between mates) and the type of organism under study (e.g., clonal organisms with physically connected or detached ramets).

Deviation from exponential relationship: Simulations showed that under isolation-by-distance on a two-dimensional grid, Moran’s \( I \) typically drops from positive values at short distances to negative values at intermediate distances before reaching values just below zero for larger distances in the absence of a cline (Epperson 2003). In our \( L. \) pulmonaria example, the variograms of molecular variance (not shown) and gene diversity showed evidence for such a humped distribution. This type of nonmonotonic autocorrelation structure is often encountered in geostatistical analysis and may arise from a periodic structure (Pyrcz and Deutsch 2003) or as a sampling artifact (Journel and Huijbregts 1978; Palmer and White 1994). It may be modeled by a dampened sine hole-effect model (Journel and Huijbregts 1978; Legendre and Legendre 1998). In addition, a clinal structure may cause a linear change of variance with distance, which can be modeled by a linear variogram model. Anisotropic variograms can help identify clinal structure, e.g., in the case of directional dispersal.

Robust variogram estimation: Testing of differences between spatial patterns from different populations is notoriously difficult, as the hypothesis concerns the underlying process and not its observed realization (Fortin et al. 2003). Many measures of spatial genetic structure suffer from a high sampling variance (Vekemans and Hardy 2004). Thus, rigorous statistical testing requires a large number of replicate populations, each with a large internal sample. Several robust variogram estimators exist (Cressie and Hawkins 1980; Cressie 1993). While Cavalli-Sforza (1984) used a robust variogram estimator based on the median variance, the \textit{modulus variogram} (Cressie and Hawkins 1980) has been shown to perform better than the basic variogram estimator in situations with at least 50% nugget variance (Cressie 1993). We will perform simulations to assess to what degree robust variogram estimators may help reducing sample size within populations.

Conclusions: Most measures of spatial genetic structure are rescaled with reference to random samples from the population. This reference is itself estimated from the data set and subject to bias unless spatial autocorrelation is accounted for. Such bias limits the interpretation of absolute values of various measures of spatial genetic structure and poses problems to the comparison between studies and to the estimation of biological parameters (Vekemans and Hardy 2004). Variogram modeling, on the other hand, estimates its reference value accounting for spatial autocorrelation, thus providing parameter estimates that are comparable between studies. Furthermore, the proposed variograms of molecular variance, gene diversity, and genetic diversity are directly interpretable without rescaling, as they provide a partitioning of genetic diversity by the distance between samples. While this article focuses on microsatellite data as interpreted under either IAM or SMM, the approach may be adapted to other types of genetic data. The formal integration with variograms makes the theory and tools of geostatistics available for population genetics, which may help to address some important challenges in bridging the gap between empirical studies of spatial genetic structure and theoretical approaches to isolation-by-distance.

We thank Magnus Nordborg and two anonymous reviewers for their helpful comments on this and an earlier version of the manuscript. This research is part of a project funded by the Swiss National Science Foundation under the National Centre of Competence in Research (NCCR) Plant Survival and through grant 3100A0-105830/1.

LITERATURE CITED


Communicating editor: M. Nordborg
APPENDIX: WORKED EXAMPLE

Example data: The example data set consists of two artificial variables $y_1$ and $y_2$ that describe the fragment lengths $x$ of two loci in $N = 6$ haploid individuals $A$–$F$ along a transect $t$. There are three multilocus genotypes $g$ with differing frequencies:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>F</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Spatial partitioning of molecular variance: The basic elements of spatial covariance are calculated as

$$
\hat{\gamma}(a, b) = \frac{1}{2}(y_a - y_b)^2.
$$

For instance, the comparison of gene copy $A$ to gene copies $B$ and $D$ provides

$$
\hat{\gamma}(A, B) = \frac{1}{2}\sum_i (y_{iA} - y_{iB})^2 = \frac{1}{2}((1 - 1)^2 + (4 - 4)^2) = 0
$$

$$
\hat{\gamma}(A, D) = \frac{1}{2}\sum_i (y_{iA} - y_{iD})^2 = \frac{1}{2}((1 - 2)^2 + (4 - 1)^2) = 5.
$$

The semivariance $\hat{\gamma}(a, b)$ for each pair of gene copies is tabulated in the following matrix:

<table>
<thead>
<tr>
<th>$\hat{\gamma}(a, b)$</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$E$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>-</td>
<td>0.5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$B$</td>
<td>0</td>
<td>-</td>
<td>0.5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$C$</td>
<td>0.5</td>
<td>0.5</td>
<td>-</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>$D$</td>
<td>5</td>
<td>5</td>
<td>4.5</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$E$</td>
<td>5</td>
<td>5</td>
<td>4.5</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>$F$</td>
<td>5</td>
<td>5</td>
<td>4.5</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

Matrix of distances $r$: Common geostatistical analysis omits distances of $r = 0$, so that an object is never compared to itself. For organisms such as the epiphytic lichen $L. pulmonaria$, however, individuals may share the same two-dimensional geographic coordinates if they grow on the same tree. Therefore, it may be important to distinguish between different individuals separated by a distance of zero in two-dimensional space and the comparison of an individual with itself:

<table>
<thead>
<tr>
<th>$r$</th>
<th>$A$</th>
<th>$B$</th>
<th>$C$</th>
<th>$D$</th>
<th>$E$</th>
<th>$F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$B$</td>
<td>1</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$C$</td>
<td>1</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>$D$</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$E$</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>$F$</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>-</td>
</tr>
</tbody>
</table>

Variogram of molecular variance: The empirical variogram of molecular variance is calculated using Equation 7, as is illustrated here for distance class $r = 2$ on the basis of unique pairs only [top or bottom triangle of matrices $\gamma(a, b)$ and $r$]:
\[
\hat{V}(2) = \frac{1}{2n_2} \sum_{2 \leq r} \sum_{(a,b) \neq 2} (\gamma_{ab} - \gamma_{ab})^2 = \frac{1}{n_2} \sum_{2 \leq r} \gamma(a, b) = \frac{1}{4} [5 + 5 + 5 + 4.5] = 4.875.
\]

The variance estimates \( \hat{V}(r) \) for all distance classes \( r \) are listed below. A weighted average of the molecular variance per distance class \( \hat{V}(r) \), weighted by \( n_r \), the number of pairs of gene copies, provides the global variance \( \hat{V} \).

<table>
<thead>
<tr>
<th>( h )</th>
<th>( \hat{V}(r) )</th>
<th>( n_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.25</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2.4375</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>4.875</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

\[\sum_r \hat{V}(r) n_r / n = 2.967 \quad 15\]

The total number of pairwise comparisons, \( n \), is given by \( n = \sum n_r = N(N - 1)/2 \).

**Variogram of gene diversity:** For the variogram of gene diversity, a dummy variable \( z_{lk} \) is defined for each allele \( k \) at each locus \( l \). The semivariance is calculated from the matrix of dummy variables following Equation 8:

For each pair of observations \( a \) and \( b \), the semivariance \( \hat{\gamma}(a, b) \) equals the number of loci at which they differ. The variogram of gene diversity is

<table>
<thead>
<tr>
<th>( z_a )</th>
<th>( z_{11} )</th>
<th>( z_{12} )</th>
<th>( z_{24} )</th>
<th>( z_{21} )</th>
<th>( \hat{\gamma}(a, b) )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
<th>( E )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>A</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>B</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
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<tr>
<td>C</td>
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<td>0</td>
<td>C</td>
<td>1</td>
<td>1</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>D</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>-</td>
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<tr>
<td>E</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>E</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
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<td>0</td>
</tr>
<tr>
<td>F</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>F</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

\[\hat{H}(r) n_r / n = 1.133 \quad 15\]

**Variogram of genotypic diversity:** For the variogram of genotypic diversity following Equation 12, a dummy variable \( z_g \) is defined for each genotype \( g \), and the semivariance is calculated from the matrix of dummy variables:

For each pair of observations \( a \) and \( b \), the semivariance \( \hat{\gamma}(a, b) \) equals the number of loci at which they differ. The variogram of genotypic diversity is

<table>
<thead>
<tr>
<th>( z_a )</th>
<th>( z_1 )</th>
<th>( z_2 )</th>
<th>( z_3 )</th>
<th>( \hat{\gamma}(a, b) )</th>
<th>( A )</th>
<th>( B )</th>
<th>( C )</th>
<th>( D )</th>
<th>( E )</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>A</td>
<td>-</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>B</td>
<td>0</td>
<td>-</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>C</td>
<td>1</td>
<td>1</td>
<td>-</td>
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<tr>
<td>D</td>
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<td>D</td>
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<tr>
<td>E</td>
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<td>1</td>
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<td>-</td>
<td>0</td>
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<tr>
<td>F</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>F</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>
As a computational shortcut, the same result can be obtained by setting all values \( \hat{\gamma}(a, b) > 0 \) to 1. The variogram of genotype diversity is

\[
\begin{array}{ccc}
  r & \hat{D}(r) & n_r \\
  0 & 0.5 & 2 \\
  1 & 0.625 & 8 \\
  2 & 1 & 4 \\
  3 & 1 & 1 \\
\end{array}
\]

\[
\sum_i \hat{D}(r) n_i/n = 0.496 
\]

**Weighting for recurrent genotypes:** Each gene copy \( a \) receives a weight \( w_a \) which is inverse to the number of gene copies of the same multilocus genotype \( g_a \). According to Equation 13, the matrix of weights \( w_a w_b \) is

\[
\begin{array}{cccccccc}
  w_a & A & B & C & D & E & F \\
  w_b & \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \Sigma w_a w_b & \\
  A & \frac{1}{2} & - & \frac{1}{2} & \frac{3}{6} & \frac{6}{6} & \frac{6}{6} & 1 & \\
  B & \frac{1}{2} & - & \frac{1}{2} & \frac{3}{6} & \frac{6}{6} & \frac{6}{6} & 1 & \\
  C & 1 & \frac{1}{2} & \frac{1}{2} & - & \frac{3}{6} & \frac{6}{6} & 2 & \\
  D & \frac{3}{6} & \frac{6}{6} & \frac{6}{6} & \frac{1}{6} & - & - & \frac{3}{6} & \\
  E & \frac{3}{6} & \frac{6}{6} & \frac{6}{6} & \frac{1}{6} & - & - & \frac{3}{6} & \\
  F & \frac{3}{6} & \frac{6}{6} & \frac{6}{6} & \frac{1}{6} & - & - & \frac{3}{6} & \\
\end{array}
\]

If \( G = 3 \) is the number of genotypes, the sum of all weights is

\[\Sigma w_a w_b = G(G - 1) = 6.\]

The variogram of molecular variance between genotypes is derived as

\[
\hat{V}(2) = \frac{1}{w_a(w_b)^{1/2}} \cdot \gamma(a, b) = \frac{1}{1/6 + 1/6 + 1/6 + 1/3} \left[ \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 5 + \frac{1}{6} \cdot 5 + \frac{1}{3} \cdot 4.5 \right] = 4.8
\]

\[
\begin{array}{ccc}
  r & \hat{V}(r) & w_s \\
  0 & 0.5 & \frac{1}{2} \\
  1 & 3.28 & \frac{1}{2} \\
  2 & 4.8 & \frac{3}{6} \\
  3 & 5 & \frac{3}{6} \\
\end{array}
\]

\[
\sum_i \hat{V}(r) w_s/w = 3.33 
\]